Problem Set: Entry Models and Simulation Estimation

Spring 2009

Due date: April 8th

This problem set deals with the dataset **tire.txt**, which has some explanatory material at the top of the file and then some data. The data come from the paper by Bresnahan and Reiss (1991) that we covered in class.

1. Your first task is to replicate the estimates for the tire dealers in Table 4, p.994 (the last column in the table). The model is as follows (please refer to the paper for details):

$$S = \text{tpop} + \lambda_1 \text{opop} + \lambda_2 \text{ngrw} + \lambda_3 \text{pgrw} + \lambda_4 \text{octy} \tag{1}$$

$$V_N = \beta_1 \text{old} + \beta_2 \text{pinc} + \beta_3 \text{lnhhd} + \beta_4 \text{ffarm} + \alpha_1 - \sum_{n=2}^{N} \alpha_n \tag{2}$$

$$F_N = \gamma_1 + \sum_{n=2}^{N} \gamma_n + \gamma_L \text{acre} \tag{3}$$

   The dependent variable is the number of tire dealers. The variable can take on the values $\{0, 1, 2, 3, 4, 5+\}$. The econometric model is an ordered probit model with a latent variable (profit) that depends on the number of active firms. The per-firm profit in an N-firm market is:

$$\Pi_N = S \cdot V_N - F_N + \epsilon \tag{4}$$

where ϵ is a market-specific error term which is distributed i.i.d. $N(0,1)$ across all markets.

Reproduce the results for the tire dealers in Table 4. Note that the unrestricted estimates of one of the α's was set to zero in the estimation reported in the paper (and you should do the same).

2. Now lets practice simulation. Adapt the previous model by changing the profit expression so that the profit of firm j in an N-firm market is:

$$\Pi_{j,N} = S \cdot V_N - F_N + \epsilon_j \tag{5}$$

where ϵ_j is a firm-specific error term which is distributed i.i.d. $N(0,1)$ across all firms. Firm j is willing to enter if $\Pi_{j,N} \geq 0 \Leftrightarrow \epsilon_j \geq -(S \cdot V_N - F_N)$.

Eliminate all markets with more than 4 firms from the sample. Use the parameter estimates from the ordered probit you estimated above. Therefore, let $N_{\max}$, the maximium number of firms in each market, be equal to 4. Given the assumptions above, we note that the vector of error terms for each of the 4 potential entrants in each market is multivariate normal, with a variance-covariance matrix equal to the identity matrix.

For each market m in the data, simulate the expected number of firms:

$$E[n_m|Z_m] = \int\int\int\int n^*_m(\epsilon_{1,m}, \epsilon_{2,m}, \epsilon_{3,m}, \epsilon_{4,m}) dF(\epsilon_{1,m}, \epsilon_{2,m}, \epsilon_{3,m}, \epsilon_{4,m},) \tag{6}$$

We assume that firm 1 moves first, followed by firm 2, firm 3, and finally by firm 4.

(a) Choose the number of simulation draws $S = 1,000$.

(b) In each draw $s = 1, \ldots, S$, generate a vector of 4 independent standard normal random variables $(\epsilon^s_1, \epsilon^s_2, \epsilon^s_3, \epsilon^s_4)$.

(c) For each vector $(\epsilon^s_1, \epsilon^s_2, \epsilon^s_3, \epsilon^s_4)$ compute the number of firms n^s_m. As discussed in class, this is a fixed point problem of solving for the largest value of n such that

$$\#\left(\epsilon_j \geq -(S \cdot V_n - F_n)\right) \geq n. \tag{7}$$

In doing this, keep in mind the stated order of entry.

(d) Compute the sample average $\frac{1}{S}\sum_{s=1}^{S} n_m^s$ to approximate the expected number of firms in market m.

3. Repeat the previous exercise, but now assume that the error terms $(\epsilon_{1,m}, \epsilon_{2,m}, \epsilon_{3,m}, \epsilon_{4,m})$ have zero mean but a variance-covariance matrix equal to

$$\Sigma = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \tag{8}$$

In order to generate a vector of multivariate normal random variables with non-zero covariances, generate a vector of independent standard normal random variables $(z_1^s, z_2^s, z_3^s, z_4^s)$ and transform them by

$$(\epsilon_1^s, \epsilon_2^s, \epsilon_3^s, \epsilon_4^s)' = \Sigma^{-\frac{1}{2}}(z_1^s, z_2^s, z_3^s, z_4^s)' \tag{9}$$

where $\Sigma^{-\frac{1}{2}}$ denotes the Cholesky factorization of the variance-covariance matrix Σ.

I recommend that you use MATLAB to do this exercise. In order to do maximum likelihood estimation, use the **fminsearch** routine to minimize the *negative* of the log-likelihood function. Use the **normrnd** function to generate normal random variables and use the **chol** routine to do Cholesky factorization.