Demand in Differentiated-Product Markets (part 1)*

Spring 2009

1 Why do demand analysis?

- If industrial organization is focused on the firm (supply-side), why spend so much time estimating demand models?
- More and more economists realize that one needs a thorough accounting of the demand-side to properly answer supply-side questions.
- Further, important determinants of firm behavior are costs, which are usually unobserved.
- A way towards estimating these costs involves an indirect approach, by estimating demand functions
  - fundamental question in IO: how much market power do firms have?
  - Market power can be measured by markups: $\frac{p-mc}{p}$.
  - If don't observe mc, what can be done?
  - recall in the monopoly case:

$$\max_p \left\{ pq(p) - C(q(p)) \right\}, \text{where } q \text{ is the demand curve} \tag{1}$$

$$\text{FOC: } \; q(p) + pq'(p) = C'(q(p))q'(p) \tag{2}$$

*These notes rely on those produced by Matthew Shum and Aviv Nevo at the optimal price, p^*,

$$\frac{p^* - mc(q(p^*))}{p^*} = -\frac{1}{\epsilon(p^*)} \tag{3}$$

where ϵ is the own-price elasticity: $q'(p^*)\frac{p^*}{q(p^*)}$.

- Can perform similar analysis for oligopoly case. This analysis crucial depends on the structure of the supply-side. If you don't believe the supply-side assumptions, then estimates are pointless.

- Start by quickly reviewing older approaches to demand estimation to motivate where the current literature stands.

2 Review of demand estimation

The Problem

The most straight-forward approach

$$q = D(p, r) \tag{4}$$

where:

- q is a J-dimensional vector of quantities demanded;

- p is a J-dimensional vector of prices;

- r is a vector of exogenous variables;

The main concern of previous work was to specify $D(\cdot)$ in a way that was both flexible and consistent with economic theory.

1. Linear Expenditure model (Stone, 1954);

2. Rotterdam model (Theil, 1965; and Barten 1966);

3. Translog model (Christensen, Jorgenson, and Lau, 1975);

4. Almost Ideal Demand System (Deaton and Muellbauer, 1980 and more recent work by Jerry Hausman).

2.1 example 1

Consider consumer's problem, where I is income,

$$\max_{x_1,x_2} U(x_1,x_2)$$
$$s.t. p_1x_1 + p_2x_2 \leq I$$

- consumer's problem yields the demand function: $x_1^*(p_1,p_2,I), x_2^*(x_1,x_2,I)$
- specify an indirect utility function:

$$V(p_1,p_2,I) = U(x_1^*,x_2^*) \tag{5}$$

- for empirical analysis, assume a particular functional form (e.g. Translog) and let k denote a consumer

$$\log(V_k(p_1,p_2,I_k)) = \alpha_0 + \sum_{i=1}^{2}\alpha_i \log(\frac{p_i}{I_k}) + \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\beta_{ij}\log(\frac{p_i}{I_k})\log(\frac{p_j}{I_k}) \tag{6}$$

  where α's and β's are to be estimated

- to estimate, need to add a stochastic term and explain how to interpret this error term.
- need to worry if price is endogenous. Firms observe error term and set price, so expect correlation between the error term and price. Need to use cost shifters (e.g. weather, transportation costs, input costs) which vary across time or geographic areas.

For differentiated products the problems with these approaches

1. Dimensionality: due to the large number of products the number of parameters will be too large to estimate.
   Example: a linear demand system, $D(p) = Ap$, where A is $J \times J$ matrix of constants, implies J^2 parameters. This problem is augmented if we attempt to use a flexible functional form (see Translog)

2. Multicollinearity/IV: In most differentiated products industries prices of the various goods will be highly collinear. This problem is augmented since we require an IV for each price. It is usually very hard to find IV that are both exogenous and will not generate moment conditions that are not nearly collinear.

3. Heterogeneity: for some applications we would like to explicitly model and estimate consumer heterogeneity.

2.2 example 2

Another example is the CES utility function

$$U(q_1, \dots, q_J) = \left(\sum_{i=1}^{J} q_i^{\rho} \right)^{\frac{1}{\rho}} \tag{7}$$

where ρ is a constant parameter that measures substitution across products. Demand is given by

$$q_k = \frac{p_k^{\frac{-1}{1-\rho}}}{\sum_{i=1}^{J} p_i^{\frac{-\rho}{1-\rho}}} I, k = 1, \dots J, \tag{8}$$

where I is the income of the representative consumer.

- Overcome dimensionality problem by imposing symmetry between different products; thus estimation involves a single parameter.

- The model implies

$$\frac{dq_i}{dp_j} \frac{p_j}{q_i} = \frac{dq_k}{dp_j} \frac{p_j}{q_k}, \text{ for all } i, k, j. \tag{9}$$

  The cross-price elasticities are restricted to be equal, regardless of the how “close” the products are in some attribute space.

- can be useful for macro or trade literature where looking at movements in the aggregate, but generally this model is inappropriate for questions of interest in IO.

3 Discrete-choice approach to modeling demand

- Solves the dimensionality problem by projecting the products onto a characteristics space (e.g. hedonics), making the relevant dimension the dimension of characteristics.

- Assume each consumer purchases at most one product in a period.

- There are N alternatives in market. Each purchase occasion, each consumer i divides her income y_i on (at most) one of the alternatives, and on an "outside good":

$$\max_{n,z} U_i(x_n, z)$$

$$s.t. p_n + p_z z = y_i$$

  where

  - x_n are chars of brand n, and p_n the price
  - z is quantity of an outside good, and p_z its price
  - often index the outside good as $n = 0$

- Substitute in the budget constraint ($z = \frac{y-p_n}{p_z}$) to derive conditional indirect utility functions for each brand:

$$U_{in}^*(p_n, p_z, y) = U_i(x_n, \frac{y_i - p_n}{p_z}) \tag{10}$$

  If outside good is bought:

$$U_{i0}^*(p_z, y) = U_i(0, \frac{y_i}{p_z}) \tag{11}$$

- Consumer chooses the brand yielding the highest cond. indirect utility:

$$\max_n U_{in}^*(p_n, p_z, y_i) \tag{12}$$

- Because only the relative utilities matter (not the absolute levels), typically consider the conditional indirect utilities normalized by the outside good's utility.
- U^*_{in} usually specified as sum of deterministic and stochastic part:

$$U^*_{in}(p_n, p_z, y_i) = V_{in}(p_n, p_z, y_i) + \epsilon_{in} \tag{13}$$

  where ϵ_{in} is observed by agent i and the firm, but not by the econometrician (this is a structural error). From agent's point of view, utility and choice are deterministic.
- Distributional assumptions on ϵ_{in}, for $n = 0, \ldots, N$, determine the form of consumer i's choice probabilities. Probability that consumer i buys brand n is:

$$D_{in}(p_1, \ldots, p_N, p_z, y_i) = Prob\big\{\epsilon_{i0}, \ldots, \epsilon_{in} : U^*_{in} > U^*_{ij} \text{ for } j \neq n\big\} \tag{14}$$

- if consumers identical, and $\big\{\epsilon_{i0}, \ldots, \epsilon_{iN}\big\}$ is iid across agents (and there are a large number of agents), then $D_{in}(p_1, \ldots, p_N, p_z, y)$ is also the aggregate market share.

3.1 Common assumptions on ϵ

1. $\big\{\epsilon_{i0}, \ldots, \epsilon_{iN}\big\}$ are distributed multivariate normal: multinomial probit. Choice probabilities do not have closed form, but they can be simulated (Keane (1994), McFadden (1989)). (e.g. GHK simulator) But difficult when there are large number of choices, because number of parameters in the variance matrix of ϵ also grows very large. Might be able to get around this problem if you can make reasonable assumptions on the variance-covariance matrix.

2. $\big\{\epsilon_{i0}, \ldots, \epsilon_{iN}\big\}$ are distributed i.i.d. type I extreme value across i:

$$F(\epsilon) = \exp\big[-\exp\big(-\frac{\epsilon - \eta}{\mu}\big)\big] \tag{15}$$

   with the location parameter $\eta = 0.577$ (Eulers constant), and the scale parameter $\mu = 1$ (usually).
   Leads to multinomial logit choice probabilities:

$$D_{in}(\cdot) = \frac{\exp(V_{in})}{\sum_{j=0,1,\ldots,N} \exp(V_{ij})} \tag{16}$$

where V_0 is often normalized to 0, (and so $\exp(V_0) = 1$). This distributional assumption on ϵ is convenient b/c there is an analytical form for the choice probabilities. The Logit model is a basis for many demand papers in empirical IO.

3.2 Problems with multinomial logit

IIA: Independence of Irrelevant Alternatives

Restrictiveness of multinomial logit: Odds ratio between any two brands j, k doesn't depend on number of alternatives available

$$\frac{D_{ij}}{D_{ik}} = \frac{\exp(V_{ij})}{\exp(V_{ik})} \tag{17}$$

Example: Red bus/blue bus problem:

- Assume that city has two transportation schemes: walk, and red bus. These 2 options provide the same expected utility (i.e. the deterministic part, V_{ij}) and so the shares are 50%, 50%. Accordingly, the odds ratio of walk/RB= 1.
- Now consider introduction of third option: train. IIA implies that odds ratio between walk/RB is still 1. Unrealistic: if train substitutes more with bus than walking, then new shares should reflect that. For example, it could be walk 45%, RB 30%, train 25%. The odds ratios for walk/RB=1.5.
- What if the third option were blue bus? IIA implies that odds ratio between walk/red bus would still be 1. Unrealistic: BB is perfect substitute for RB, so that new shares are walk 50%, RB 25%, BB 25%, and odds ratio walk/RB=2
- This feature of logit models is especially troubling if you want to use the logit model to predict the market share of new products.

Overcoming IIA Overcoming IIA

1. nested logit: assume a particular correlation structure amoung $\{\epsilon_{i0}, \ldots, \epsilon_{iN}\}$. Within nest brands are "closer substitutes" than across-nest brands (see Goldberg 1995 – draw graph).

2. Another way to think about the nested logit. Indirect utility is:

$$u_{ij} = x_j\beta - \alpha p_j + \eta_{ij} \tag{18}$$

$$u_{ij} = \delta_j + \eta_{ij} \tag{19}$$

   where η_{ij} is the taste-for-variety error term. Lets put some structure on η. Divide products into G mutually-exclusive groups (so there are $G+1$ total, including the outside good). Want same products in a group (or nest) to have a similar unobserved shock, or

$$u_{ij} = \delta_j + \zeta_{ig} + (1-\sigma)\epsilon_{ij}, \tag{20}$$

   where ϵ is distributed iid type 1 extreme value and ζ is common to all products in group $g = 0, 1, \ldots, G$ for consumer i. $\sigma \in [0, 1]$ is a parameter to be estimated (i.e. nesting parameter). ζ is assumed to have a distribution such that $[\zeta + (1-\sigma)\epsilon]$ is distributed type 1 extreme value.

3. Note that as $\sigma \to 1$ then within group correlation of utility values goes to one, while as $\sigma \to 0$ the within group correlation goes to zero.

4. random coefficients: assume the logit model, but for agent i:

$$U^*_{in} = X_n\beta_i - \alpha_i p_n + \epsilon_{in} \tag{21}$$

   note: the coefficients are agent-specific.

   Then aggregate market share is

$$\int D_{in}(p_1, \ldots, p_N, p_z, X; \alpha_i, \beta_i) dF(\alpha_i, \beta_i), \tag{22}$$

   which differs from the individual choice probability. The elasticity implications of IIA disappear. We will study the random coefficients model in detail.

5. Important distinction between nested and random coef. logit: NL implies that IIA disappears at the individual level, while RC implies that IIA disappears only at the aggregate level.